Product innovation, sell-off, and entry deterrence

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Abstract

The paper shows that there may be greater or lesser incentive to develop a substitute when faced with the threat of entry. The fear that product innovation may lead to entry, which can only be forestalled by sell-off, may make an incumbent less likely to innovate. On the other hand, the incumbent may have more incentive to engage in product innovation and sell-off in order to deter entry if the entrant expects its entry to force the incumbent to develop a substitute and exit from the current product market. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

When a monopolist decides whether or not to engage in product innovation, it naturally considers the effect of product innovation on entry. We compare the incentive of a monopolist to engage in product innovation of a higher quality substitute with and without the threat of entry.¹ No firm other than the incumbent

¹The examples of the demands used in this paper show that the substitute is of higher quality than the current product. But the results of this paper can hold even when the substitute is not of higher quality than the current product.
can develop a substitute. We further consider two cases when there is a potential entrant. First, product innovation may take so long that an entrant cannot expect its entry to force the incumbent to develop a substitute and exit from the current product market any time soon (‘Slow Innovation’ model). In this scenario, the incumbent cannot finish product innovation before production if it is started after entry. Second, product innovation may take little time so that the incumbent can finish product innovation and exit from the current product market quickly (‘Quick Innovation’ model). In this scenario, the incumbent can finish product innovation before production, even when it is started after the entrant decides whether or not to enter.

Entry deterrence requires some precommitment of the incumbent for aggressive postentry behavior. The list of the means for precommitment considered in the literature for entry deterrence includes capital (Spence, 1977; Dixit, 1979, 1980), brand proliferation (Schmalensee, 1978), and divisionalization (Schwarz and Thompson, 1986). As Judd (1985) noted, credibility is essential for the entry-deterring precommitment. In our Slow Innovation model, the incumbent can deter entry either by not engaging in innovation or by engaging in innovation and sell-off. In the Quick Innovation model, not engaging in innovation before entry cannot precommit aggressive postentry behavior because the incumbent can develop a substitute and exit from the current product market soon after entry.

In both the Slow Innovation model and the Quick Innovation model, in equilibrium entry is not accommodated. This result accords with the no-entry result in Schwarz and Thompson (1986). In both the Slow Innovation model and the Quick Innovation model, there is a case in which the incumbent develops a substitute and sells off one product line to deter entry when it does not sell off any product lines without the threat of entry. In this case, we can say that entry is deterred by sell-off. In both models, there is also a case in which entry is deterred by not engaging in product innovation. In the Quick Innovation model, there is a case in which entry is deterred by innovation and sell-off.

There is a large literature about entry deterrence and about innovation. But there are not many studies which analyze the incentive for innovation when innovation can affect entry decision. Gilbert and Newbery (1982) analyze how the existence of a potential innovator affects the incumbent’s incentive to develop a substitute when entry to the current product market is blocked. They find that the incumbent may engage in preemptive patenting. The main differences between their model and ours concern the possibility of entry and innovation ability. In Gilbert and Newbery’s model, the potential entrant cannot enter the market where the incumbent is operating, but both the incumbent and potential entrant can

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2 This assumption can be justified in situations in which product innovation needs a certain amount of know-how obtained through learning by doing.

3 Divisionalization in Schwarz and Thompson (1986) is essentially the same as sell-off in our model. But it is somewhat doubtful that a divisionalized firm does not maximize its total profit in making an exit decision of divisions as well as pricing. Thus, our model considers sell-off of a product line instead of divisionalization.
develop a substitute. A patent by one firm makes further entry impossible in the substitute market. In our model, the potential entrant can enter the market where the incumbent is now operating, but only the incumbent can develop a substitute.

The rest of this paper is organized as follows. Section 2 analyzes the incumbent’s decision for product innovation and sell-off when there is no potential entrant and when there is a potential entrant and product innovation takes a long time. In Section 3, we analyze the case in which there is a potential entrant and product innovation does not take a long time. Section 4 concludes the paper.

2. Slow innovation

An incumbent monopolist produces a product, and there is a potential entrant which can produce the same product by incurring entry cost \( C_e \) \((C_e > 0)\). Assume that there is no production cost. Furthermore, we assume that when the entrant enters, the two firms will engage in Bertrand competition. Because of the cut-throat price competition, only one firm will produce the current product in equilibrium. Thus, we assume that there is only one potential entrant. Furthermore, we assume that there is a negligible but positive exit cost \( e \). Then a firm will exit the market if and only if its profit is negative.

Now let us assume that the technology to produce a higher quality substitute becomes available to the incumbent. We will call the existing product the low-quality product and the newly available substitute the high-quality product. The incumbent can produce the high-quality product with constant unit cost \( c \) \((c > 0)\) after incurring innovation cost \( C_p \) \((C_p > 0)\). We assume that the technology to produce the high-quality product is not available to the entrant.

Let \( L \) denote a firm that produces only the low-quality product, \( H \) denote a firm that produces only the high-quality product, and \( LH \) denote a firm that produces both products. We refer to a profile of firms as market structure. Because we will allow the incumbent to sell off one product line after product innovation, the set of relevant market structures is \{\( (L), (H), (LH), (L, L), (L, H), (L, LH), (L, L, H) \}\}. We refer to revenue minus production cost as (gross) profit. That is, neither innovation cost nor entry cost is subtracted when calculating profit. Let \( \pi^j_i \) denote the profit of firm \( i \) in market structure \( j \) in the Nash equilibrium of the price setting game. When the market structure \( j \) is a monopoly, we drop the superscript \( i \) for the purpose of simplicity.

We make the following assumptions on equilibrium profits:

\[
\begin{align*}
\pi^L_{L,H}, \quad \pi^H_{L,H} &> 0; \\
\pi_L &> \pi^L_{L,H} ; \\
\pi_{LH} &> \max\{\pi_L, \pi_H, \pi^L_{L,H} + \pi^H_{L,H}\}; \\
\end{align*}
\]

(A1) (A2) (A3)
\[ \pi_{L,L}^t = \pi_{L,H,L}^t = \pi_{L,L,H}^t = 0; \quad (A4) \]

\[ \pi_{L,L,H}^H < \pi_{L,H}^H. \quad (A5) \]

(A1) says that when the market structure is (L, H), both firms earn positive profit. (A2) states that if a firm produces only the low-quality product, its profit is greater when there is no other firm than when there is another firm producing the high-quality product. (A3) says that producing both products is better for the monopolist than producing only one product or selling off. (A4) states that when there are two firms producing the low-quality product, the firm which produces only the low-quality product earns zero profit. (A5) says that when a firm produces only the low-quality product, the other firm earns more profit when it produces only the high-quality product than when it produces both products.

To justify assumptions (A1)–(A5), we construct two examples with specific demands for the products. We assume that a continuum of consumers of measure one exists, and each consumer purchases one unit of the low-quality product, or one unit of the high-quality product, or none. Each consumer’s utility is 0 if he does not purchase any product, \( v_L - p_L \) if he purchases the low-quality product at price \( p_L \), and \( v_H - p_H \) if he purchases the high-quality product at price \( p_H \). Assume that \( v_L \) is uniformly distributed on \([0, 1]\). We consider two distributions of \( v_H \). The first is Demand 1, in which \( v_H = 2v_L \), and the second is Demand 2, in which \( v_H = 1 \). We assume that \( 0 < c < 1 \). Table 1 gives the profits in the Nash equilibrium of the price setting game with Demand 1 and Demand 2, whose derivation can be found in Appendix A. The equilibrium profits in Table 1 satisfy the assumptions.

<table>
<thead>
<tr>
<th>Market structures</th>
<th>Payoffs with Demand 1</th>
<th>Payoffs with Demand 2</th>
</tr>
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<tbody>
<tr>
<td>(L)</td>
<td>( \pi_L = \frac{1}{4} )</td>
<td>( \pi_L = \frac{1}{4} )</td>
</tr>
<tr>
<td>(H)</td>
<td>( \pi_H = \frac{(2 - c)^2}{8} )</td>
<td>( \pi_H = 1 - c )</td>
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<tr>
<td>(LH)</td>
<td>( \pi_{L,H} = \frac{1 + (1 - c)^2}{4} )</td>
<td>( \pi_{L,H} = \frac{(2 - c)^2}{4} )</td>
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<tr>
<td>(L, L)</td>
<td>( \pi_{L,L}^L = 0 )</td>
<td>( \pi_{L,L}^L = 0 )</td>
</tr>
<tr>
<td>(L, H)</td>
<td>( \pi_{L,H}^L = \frac{2(1 + c)^2}{49} )</td>
<td>( \pi_{L,H}^L = \min \left{ \frac{(1 + c)^2}{9}, \frac{1}{4} \right} )</td>
</tr>
<tr>
<td>(L, H)</td>
<td>( \pi_{L,H}^H = \frac{(4 - 3c)^2}{49} )</td>
<td>( \pi_{L,H}^H = \min \left{ \frac{(2 - c)^2}{9}, \frac{1 - c}{2} \right} )</td>
</tr>
<tr>
<td>(L, LH)</td>
<td>( \pi_{L,L,H}^L = 0, \pi_{L,L,H}^H = \frac{(1 - c)^2}{4} )</td>
<td>( \pi_{L,L,H}^L = 0, \pi_{L,L,H}^H = \frac{(1 - c)^2}{4} )</td>
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<td>(L, L, H)</td>
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<td>( \pi_{L,L,H}^L = 0, \pi_{L,L,H}^H = \frac{(1 - c)^2}{4} )</td>
</tr>
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</table>
We use both Demand 1 and Demand 2 because they are distinct in the sense that \( \pi^H_{L,H} > \pi_{L,H} - \pi_L \) with Demand 1 and \( \pi^H_{L,H} < \pi_{L,H} - \pi_L \) with Demand 2.

If the incumbent decides to engage in product innovation, it must choose whether or not to sell off one product line. If it sells off, the two firms will engage in price competition. We assume that the incumbent has bargaining power in selling off one product line, so that the incumbent’s payoff when it chooses sell-off is the sum of the equilibrium profit of the two firms net of the innovation cost \( C_p \).

Thus, it is irrelevant which product line the incumbent sells off. It is also assumed that sell-off is irreversible.

First, we consider the case in which there is no potential entrant. In such a case, the incumbent will never sell off one product line after innovation due to (A3). It chooses either not to engage in product innovation and being \( L \) or to engage in product innovation and being \( LH \). Proposition 1 gives the incumbent’s optimal decision when there is no entrant. Let \( N \) denote the decision not to engage in product innovation and \( P \) denote engaging in product innovation.

**Proposition 1.** The incumbent chooses \( N \) if \( C_p > \pi_{L,H} - \pi_L \), \( N \) or \( P \) if \( C_p = \pi_{L,H} - \pi_L \), and \( P \) otherwise.

All the proofs of propositions and lemmas in this paper are omitted since they are simple comparisons using the assumptions (A1)–(A5).

Now, we consider the case in which there is a potential entrant. After the incumbent decides whether or not to engage in product innovation and whether or not to sell off, the potential entrant decides whether or not to enter. After the entrant makes its decision, the incumbent can exit from any product line. We do not allow the incumbent to engage in product innovation after it has decided not to and the potential entrant decides whether or not to enter.

Fig. 1 shows the game tree. At node I1, the incumbent chooses one of three actions: \( N \), \( P \), and \( PS \). \( N \) denotes not engaging in product innovation, \( P \) denotes engaging in product innovation, and \( PS \) denotes engaging in product innovation and selling off one product line. If the incumbent chooses \( N \) or \( PS \), the potential entrant will never enter the market because the entrant will earn zero profit in the ensuing Bertrand competition. Therefore, the game ends without entry if the incumbent chooses \( N \) or \( PS \).

If the incumbent chooses \( P \) at node I1, then the game proceeds to subgame \( G_p \). In \( G_p \), at node E1, the entrant chooses one of two actions: \( n \) and \( e \). Here, \( n \) denotes staying out and \( e \) denotes entry. If the entrant chooses \( n \), the incumbent will never exit and/or sell off.

If the entrant chooses \( e \), then at node I2, the incumbent chooses one of three actions: \( L \), \( H \), \( LH \). \( L \) denotes the incumbent’s staying only in the low-quality product market, that is it exits from the high-quality product market. \( H \) denotes the incumbent’s staying only in the high-quality product market, that is it exits from the low-quality product market. \( LH \) denotes the incumbent’s staying in both markets. We do not consider the incumbent’s decision of sell-off at node I2.
because it is not optimal to the incumbent, regardless of whether or not the entrant enters. If the potential entrant does not enter, sell-off is dominated by LH. If it does enter, sell-off is dominated by H. We also do not consider the entrant’s decision of exit because exit after entry is dominated by staying out.

Now, we will derive the subgame perfect equilibrium (SPE) of the game in Fig. 1. Lemma 1 derives SPE moves at each node, and Proposition 2 characterizes the SPE of the game. In all of the lemmas and propositions, statements refer to characteristics of SPE. For simple exposition, we omit the expression ‘in an SPE’ in the lemmas and propositions.

**Lemma 1.** At node I2, the incumbent chooses H. At node E1, the entrant’s choice is n if \( C_E > \pi_{L,H}^H \), e or n if \( C_E = \pi_{L,H}^L \), and e otherwise.

**Proposition 2.** The equilibrium, as a function of parameter values, is as shown in Fig. 2(b).

The incumbent never accommodates entry because accommodation is domi-
nated by innovation and sell-off. Moreover, there will be no entry if the incumbent does not innovate. When $C_p < (>) \pi_{LH} - \pi_L$, the incumbent prefers to innovate (not to innovate) if innovation does not induce entry. Thus, when $C_p > \pi_{LH} - \pi_L$, the incumbent does not innovate. When $C_e > (>) \pi^L_{LH}$, innovation does not (does) induce entry. Thus if $C_p < \pi_{LH} - \pi_L$ and $C_e > \pi^L_{LH}$, the incumbent innovates. When $C_p < \pi_{LH} - \pi_L$ and $C_e < \pi^L_{LH}$, the incumbent either chooses not to innovate or to innovate and sell off. In this case, the incumbent chooses not to innovate (to innovate and sell off) if $C_p > (>) \pi^L_{LH} + \pi^H_{LH} - \pi_L$.

With Demand 2, $\pi^L_{LH} + \pi^H_{LH} - \pi_L > 0$. With Demand 1, $\pi^L_{LH} + \pi^H_{LH} - \pi_L > 0$ if and only if $c < \hat{c} = 0.35$. Thus, the region for PS may not exist if $c > \hat{c}$ with Demand 1.

Fig. 2(b) shows that entry is never accommodated. This result agrees with the no-entry result in Schwarz and Thompson (1986). Comparing (a) and (b) in Fig. 2, we find that if $C_p > \pi_{LH} - \pi_L$, the incumbent does not engage in product innovation and entry is blocked. If $C_p < \pi_{LH} - \pi_L$ and $C_e > \pi^L_{LH}$, then the incumbent engages in product innovation and entry is blocked.

When $C_p < \pi_{LH} - \pi_L$ and $C_e < \pi^L_{LH}$, entry is deterred. The incumbent can deter entry either by not engaging in product innovation or by engaging in product innovation and sell-off. If $\pi^L_{LH} + \pi^H_{LH} - \pi_L < C_p < \pi_{LH} - \pi_L$ and $C_e < \pi^L_{LH}$, the incumbent does not engage in product innovation to deter entry, though it does engage in product innovation in the entry-free situation. In this case, the incumbent stays lean and hungry, in the terminology of Fudenberg and Tirole (1984), by not engaging in product innovation.

If $C_p < \pi^L_{LH} + \pi^H_{LH} - \pi_L$ and $C_e < \pi^L_{LH}$, the incumbent engages in product innovation and sells off one product line to deter entry. Because the incumbent engages in product innovation in the entry-free situation if $C_p < \pi_{LH} - \pi_L$, we say...
that in this case, entry is deterred by sell-off rather than by product innovation and sell-off.

3. Quick innovation

In this section, we analyze the Quick Innovation model. Fig. 3 shows the game tree. At node I_1, the incumbent chooses one of three actions: N, P, and PS. If the incumbent chooses N, then the game proceeds to a subgame $G_N$. If the incumbent chooses P or PS, then the game proceeds as in the Slow Innovation model. In $G_N$,
at node E2, the entrant chooses one of two actions: n and e. Here, n denotes staying out and e denotes entry. After the entrant chooses n or e, the incumbent chooses one of three actions: N, P, and PE. N denotes not engaging in product innovation and being L, P denotes engaging in product innovation and being LH, and PE denotes engaging in product innovation and being H by exiting from the low-quality product market.

Now, we will derive the SPE of the game in Fig. 3. Lemma 2 derives SPE moves at each node, and Proposition 3 characterizes the SPE of the game.

**Lemma 2.** At node I3, the incumbent chooses N if \( C_p > \pi_{LH} - \pi_L \), N or P if \( C_p = \pi_{LH} - \pi_L \), and P otherwise. At node I4, the incumbent chooses N if \( C_p \geq \pi_{LH}^H \), and PE otherwise. At node E2, the entrant chooses n if \( C_p \geq \pi_{LH}^H \) or \( C_k > \pi_{LH}^L \), n or e if \( C_p < \pi_{LH}^H \) and \( C_k = \pi_{LH}^L \), and e otherwise.

**Proposition 3.** The equilibrium, as a function of parameter values, is as shown in Fig. 4(b).

As in the Slow Innovation model, the incumbent never accommodates entry because accommodation is dominated by innovation and sell-off. But in the Quick Innovation model, not engaging in innovation cannot always deter entry. This point makes the equilibrium in the Quick Innovation model distinct from that in the Slow Innovation model. In the Quick Innovation model, the incumbent can (cannot) deter entry by not engaging in innovation if \( C_p > (\leq) \pi_{LH}^H \).

(a) Equilibrium with Demand 1

(b) Equilibrium with Demand 2

Fig. 4.
When \( C_p > \pi_{L_H}^L \), there will be no entry regardless whether the incumbent innovates or not. In this case, the incumbent does not (does) innovate if \( C_p > (\leq) \pi_{L_H} - \pi_L \). When \( C_p < \pi_{L_H}^L \) and \( C_p < \pi_{L_H}^H \), the only way for the incumbent to deter entry is to innovate and sell off and the incumbent chooses it. When \( C_p < \pi_{L_H}^L \) and \( C_p > \pi_{L_H}^H \), the incumbent can deter entry either by not engaging in innovation or engaging in innovation and sell-off. In this case, the incumbent chooses entry deterrence by innovation and sell-off.

Fig. 4 shows that entry is never accommodated. Comparing Fig. 2 and Fig. 4, we can see that because of (A2), the region for engaging in product innovation and selling off is wider in the Quick Innovation model than in the Slow Innovation model. If \( C_p < \pi_{L_H}^H \), the incumbent is supposed to engage in product innovation and exit from the low quality product market after entry. Thus if \( C_p < \pi_{L_H}^H \), the incumbent cannot deter entry by not engaging in product innovation in the Quick Innovation model. Therefore, if \( C_p < \pi_{L_H}^L \) and \( \pi_{L_H}^L + \pi_{L_H}^H - \pi_L < C_p < \min\{\pi_{L_H}^H, \pi_{L_H}^L - \pi_L\} \), the incumbent deters entry by sell-off in the Quick Innovation model, though it prefers to deter entry by not engaging in product innovation as in the Slow Innovation model.

Fig. 4(a) shows that with Demand 1, if \( C_p < \pi_{L_H}^L \) and \( \pi_{L_H} - \pi_L < C_p < \pi_{L_H}^H \), the existence of an entrant forces the incumbent to engage in product innovation and sell-off, though it does not engage in product innovation in the entry-free situation. In this case we can say that entry is deterred by product innovation and sell-off.

4. Conclusions

This paper analyzed the decision of a monopolist for product innovation with and without the threat of entry. We considered two cases in which there is a potential entrant. One is the case in which product innovation takes so long that the entrant cannot expect its entry to force the incumbent to engage in product innovation and exit from the current product market any time soon (Slow Innovation model). The other case is one in which product innovation takes little time so that the incumbent can finish product innovation and exit from the current product market quickly (Quick Innovation model).

In both the Slow Innovation model and the Quick Innovation model, the incumbent never accommodates entry because accommodation is dominated by innovation and sell-off. In the Slow Innovation model, there will be no entry if the incumbent does not innovate. But in the Quick Innovation model, not engaging in innovation cannot always deter entry because the incumbent may innovate and exit from the current product market when entry takes place. This point makes the equilibrium in the Quick Innovation model distinct from that in the Slow Innovation model. In both the Slow Innovation model and the Quick Innovation model, there is a case in which the incumbent deters entry by not engaging in innovation. In both models, there is also a case in which the incumbent develops a substitute and sells off one product line to deter entry when it does not sell off any
product lines without the threat of entry (entry deterrence by sell-off). In the Quick Innovation model, there is a case in which the incumbent develops a substitute and sells off one product line to deter entry when it does not innovate without the threat of entry (entry deterrence by innovation and sell-off).

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**Appendix A**

Because there is a continuum of consumers, a single consumer’s decision does not affect the profits of firms. For simplicity, we assume that when a buyer is indifferent between buying and not buying a product, he buys it. We also assume that when a buyer is indifferent between buying a high-quality product and buying a low-quality product, he buys the high-quality product. Furthermore, we assume that if two firms set the same price for the low-quality product, then the market demand for the low-quality product is evenly divided between the two firms. We refer to the consumer whose utility is $v_L - p_L$ when he buys a low-quality product as consumer $v_L$.

**Equilibrium of Price Setting Game with Demand 1:**

1. (L) When firm L sets price $p_L$, consumer $v_L$ buys its product if and only if $v_L - p_L \geq 0$. Thus its profit is $p_L \max \{1 - p_L, 0\}$. To maximize profit, L sets $p_L = 1/2$. Thus $\pi_L = 1/4$.

2. (H) When firm H sets price $p_H$, consumer $v_H$ buys its product if and only if $2v_L - p_H \geq 0$. Thus its profit is $(p_H - c) \max \{1 - p_H / 2, 0\}$. To maximize profit, H sets $p_H = 1 + c / 2$. Thus $\pi_H = (2 - c) / 8$.

3. (LH) When firm LH sets price $p_L$ for the low-quality product and price $p_{HL}$ for the high-quality product, consumer $v_L$ buys the low-quality product if $v_L - p_L \geq 0$, the high-quality product if $2v_L - p_{HL} \geq 0$, and does not buy any product otherwise. Thus its profit is $p_L \max \{\min \{p_{HL} - 2p_L, 1 - p_L\}, 0\} + (p_H - c) \max \{\min \{1 - (p_H - p_L), 1 - p_H / 2\}, 0\}$. To maximize profit, LH sets $p_L = 1/2$ and $p_{HL} = 1 + c / 2$. Thus $\pi_{LH} = (1 + (1 - c)^2) / 4$.

4. (L, L) Bertrand competition between the two firms makes the price 0 in the unique Nash equilibrium. Thus $\pi_{L,L} = 0$.

5. (L, H) When firm L sets price $p_L$ and firm H sets price $p_{HL}$, L’s profit is $p_L \max \{\min \{p_{HL} - 2p_L, 1 - p_L\}, 0\}$ and H’s profit is $(p_{HL} - c) \max \{\min \{1 - (p_{HL} - p_L), 1 - p_{HL} / 2\}, 0\}$. Then in the unique Nash equilibrium, $p_L = (1 + c) / 7$ and $p_{HL} = 4(1 + c) / 7$. Thus $\pi_{L,H} = 2(1 + c)^2 / 49$ and $\pi_{H,L} = (4 - 3c)^2 / 49$.

6. (L, LH) Bertrand competition between the two firms makes the price 0 in
the low-quality product market in Nash equilibria. When the price in the low-quality market is 0 and firm LH sets price $p_H$ for the high-quality product, LH’s profit is $(p_H - c)\max\{1 - p_H, 0\}$. Because firm LH maximizes its profit given firm L’s price in Nash equilibria, $p_H = (1 + c)/2$ in the unique Nash equilibrium. Thus $\pi_{L,H}^L = 0$ and $\pi_{L,H}^{LH} = (1 - c)^2/4$.

(7) (L, L, H) Bertrand competition between the two firms in the low-quality product market makes the price 0 in the low-quality product market in Nash equilibria. When the price in the low-quality market is 0 and firm H sets price $p_H$ for the high-quality product, H’s profit is $(p_H - c)\max\{1 - p_H, 0\}$. Because firm H maximizes its profit given other firms’ prices in Nash equilibria, $p_H = (1 + c)/2$ in the unique Nash equilibrium. Thus $\pi_{L,L,H}^H = 0$ and $\pi_{L,L,H}^{LL} = (1 - c)^2/4$.

Equilibrium of Price Setting Game with Demand 2:

(1) Equilibria in (L), (L, L), (L, LH), and (L, L, H) are the same as with Demand 1.

(2) (H) When firm H sets price $p_H$, consumers buy its product if and only if $1 - p_H > 0$. Thus its profit is $(p_H - c)$ if $p_H < 1$ and 0 otherwise. To maximize profit, H sets $p_H = 1$. Thus $\pi_H^H = 1 - c$.

(3) (LH) When firm LH sets price $p_L$ for the low-quality product and price $p_H$ for the high-quality product, consumer $v_L$ buys the low-quality product if $v_L - p_L > 1 - p_H$ and $v_L - p_L \geq 0$, the high-quality product if $1 - p_H \geq \max\{v_L - p_L, 0\}$, and does not buy any product otherwise. Thus its profit is $p_L \max\{p_H - p_L, 0\} + (p_H - c)\min\{1 - (p_H - p_L), 1\}$ if $p_H \leq 1$ and $p_L \max\{1 - p_L, 0\}$ otherwise. To maximize profit, LH sets $p_L = 1 - c/2$ and $p_H = 1$. Thus $\pi_{L,H}^L = (2 - c)^2/4$.

(4) (L, H) When firm L sets price $p_L$ and firm H sets price $p_H$, L’s profit is $p_L \max\{p_H, 1 - p_L, 0\}$. H’s profit is $(p_H - c)\min\{1 - (p_H - p_L), 1\}$ if $p_H \leq 1$ and 0 otherwise. Then in the unique Nash equilibrium, $p_L = \min\{(1 + c)/3, 1/2\}$ and $p_H = \min\{2(1 + c)/3, 1\}$. Thus $\pi_{L,H}^L = \min\{(1 + c)^2/9, 1/4\}$ and $\pi_{L,H}^H = \min\{(2 - c)^2/9, (1 - c)/2\}$.

References


