系所別：資訊工程學系，網路學習科技研究所　　科目：離散數學

1. (20%) Calculate the following summations where n, r, an s are positive integers
   (a) $\sum_{k=0}^{\infty} C(n, k)$
   (b) $\sum_{k=0}^{\infty} (-1)^k C(n, k)$
   (c) $\sum_{k=0}^{\infty} k\, C(n, k)$
   (d) $\sum_{k=0}^{\infty} C(n, k)^2$
   (e) $\sum_{k=0}^{n} C(k, r)$
   (f) $\sum_{j=0}^{r} C(n-j, r-j)$
   (g) $\sum_{k=0}^{\infty} C(r, k)C(s, n-k)$

2. (20%) Find the logical equivalence relations among the following compound statements
   (a) $\neg p \vee q \vee r$
   (b) $p \vee \neg q \vee r$
   (c) $p \vee q \vee \neg r$
   (d) $(p \rightarrow q) \rightarrow r$
   (e) $p \rightarrow (q \rightarrow r)$
   (f) $\neg p \vee q \vee \neg r$
   (g) $p \wedge q \vee \neg r$
   (h) $(p \wedge q) \rightarrow r$
   (i) $p \vee \neg q \vee \neg r$
   (j) $\neg p \vee \neg q \vee r$

參考用

3. (a) (10%) Find the smallest positive integer g such that $204m + 330n = g$ for some integers m and n.
   (b) (10%) Find the inverse of 34(mod 89); that is an integer a such that $a \times 34 \pmod{89}$ is equal to 1. Please do not just give the answer and clearly describe your process to get the answer.

4. (a) (10%) Find a recurrence relation for a_n, the number of ways a sequence of 1's and 3's can sum to n. For example, $a_4 = 3$ since 4 can be obtained with the following sequences: 1111 or 13 or 31. Let α be the real number such that $\lim_{n\to\infty} a_n/\alpha^n = 1$. Estimate the value of α with 1 digit to the right of the decimal point.
   (b) (10%) Find and solve a recurrence relation for the number of n-digit quinary sequences that have an even number of 0's (quinary sequences using only the digits 0, 1, 2, 3, 4). Use your result to compute the number when $n = 100$.

注意：背面有試題